# Precalculus
# Curriculum Framework

Mapping High School Precalculus to:

*Mathematics Cognitive Demands*
*Texas State Standards: Texas Essential Knowledge and Skills, TEKS*
*Underlying Processes in TEKS assessed in*
*Texas Assessment of Knowledge and Skills, TAKS*
*National Council of Teachers of Mathematics, NCTM,*
*Principles and Standards 2000*

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**National Science Foundation**

**The El Paso Collaborative for Academic Excellence**

# Curriculum Framework for High School Precalculus

A group of classroom K-16 mathematics teachers, faculty, curriculum specialists and department chairs met over the course of a year and developed a curriculum framework for high school Precalculus, an important step in developing explicit and comprehensive goals for teachers of Precalculus in the El Paso area. This framework represents the collective work of classroom teachers from K-12 schools, and faculty from El Paso Community College and the University of Texas at El Paso. It is meant to assist mathematics teachers in ensuring that current high school courses are aligned with the first year college mathematics course that entering college freshmen will take. Precalculus was the last course to be developed in the mathematics alignment process. Previously, the group developed frameworks for Algebra II, Algebra I, Geometry, and K-8 Mathematics. The expectation is that as teachers use the frameworks to provide challenging courses and curriculum in mathematics in high school, the number of students having to enroll in remedial mathematics courses in college, will be reduced. Students will benefit because of the collective effort of teachers K-16 who will embrace the next stage in this process: implementation with the goal of providing practical revision. With participation from every high school mathematics teacher, curriculum frameworks will become the standard in mathematics coursework for every student in El Paso.

**Background**

In 1998, the El Paso Collaborative board identified as its top priorities: 1) continuing to focus on mathematics, and 2) the alignment of the mathematics curriculum, K-16. A review of local data on mathematics achievement showed a larger number of students enrolled in and completing college preparatory mathematics courses in high school. It also revealed a continuous increase in student achievement on TAAS. This higher student achievement, however, did not reflect student readiness for college mathematics courses. Further review of the data revealed that large numbers of high school students were placing and enrolling in remedial courses at El Paso Community College and the University of Texas at El Paso. While many factors contribute to the placing of large numbers of students in remedial courses, one known factor is that there was little alignment between what high school teachers expect students to know and be able to do and the expectations of college and university faculty.

To deal with some of these issues, the El Paso Collaborative for Academic Excellence proposed and was funded, by the National Science Foundation and the Pew Charitable Trusts, to support a K-16 Mathematics Alignment Initiative to align mathematics curriculum, instruction, and assessment. A beginning goal of the Initiative was to determine what students need to know and be able to do in a high school mathematics course that would prepare them to enroll in and successfully complete a college level, mathematics course. Algebra II was identified as the pivotal course that could provide high school students with preparation for entering and successfully completing a college freshman precalculus course without first needing remediation.

**Working Group**

The Initiative convened a working group of K-16 classroom teachers and faculty to write a framework for Algebra II that teachers could utilize as a curriculum guide, no matter what instructional materials they were using for the course. The Working Group included: K-12 classroom teachers from both urban and rural independent school districts; mentors with specialization in mathematics from the three major school districts; mathematics instructors from El Paso Community College; and professors representing the Colleges of Education (mathematics education), Science (mathematics) and Engineering (computer science) from the University of Texas at El Paso. A complete list of participants in the Working Group is attached.

To prepare for writing the curriculum framework for Precalculus, as with all the others, the group engaged in dialogue and discussion focused on mathematics teaching and learning. Using formatted discussions, the group:

- analyzed and discussed student performance in mathematics using data collected from state mandated assessments, and college placement tests;
- examined textbooks, course requirements, outline format, and state and national placement tools used to assess student knowledge of mathematics;
- reviewed the Texas Essential Knowledge and Skills (TEKS) and the National Council of Teachers in Mathematics (NCTM) Principles and Standards 2000;
- discussed how concepts were connected and developed at different grade levels, and how they led to concepts in higher mathematics;
- discussed international education systems, mathematics teaching and learning, and other issues related to mathematics education in other countries, such as Germany, Japan, Mexico, and Russia;
- identified alternate ways of assessing student learning that provide for standards-based assessment;
- discussed models of teaching mathematics; and
- reviewed and discussed literature on mathematics education.

After these initial meetings, the group met bimonthly during academic years and for several days in the summer, to write curriculum frameworks for Algebra II, and continued with K-8 Mathematics, Algebra I, Geometry, and Precalculus. Content for the high school course was placed in text outline form as well as matrix form to map content topics to cognitive demands. Course content was also mapped to textbooks and materials being used in the three major independent school districts, as well as to state (TEKS) and national (NCTM) mathematics standards. A table on standards-based assessment was attached to the matrix as a guide for assessing student learning and understanding of mathematics content. Also included are suggestions on how to determine a student's grade for the course and a timeline for covering the course.

**K-16 Leaders Group**

A leaders group, that included district leaders and central office personnel from the three major independent school districts, the provost of the University, science and education deans and mathematics department chairs from both the University and Community College, as well as lead principals and teachers from the districts, was also convened to dialogue and discuss and issues in mathematics education. This group reviewed the curriculum framework several times, each time providing guidance and feedback as it was being developed.

**Needs**

What we need now is assistance from high school principals and teachers who will review, revise and make practical use of the framework during the current academic year. Ideally, the curriculum frameworks should be reviewed by every high school teacher of Algebra I, Geometry, Algebra II, and Precalculus to help prioritize aligning mathematics curriculum, instruction, and assessment, K-16. In order to continue our work, we need active participation from every mathematics department in every school in both urban and rural independent school districts and by postsecondary mathematics instructors and chairs.

Call 747-5778 for more information on how you can be involved in this important work of reviewing and revising these frameworks

*Lucy H. Michal*
*Director, K-16 Mathematics Alignment Initiative*
*MSP Director of Mathematics and Science*
*lmichal@utep.edu*

# K-16 MATHEMATICS ALIGNMENT WORKING GROUP

| | | | |
|---|---|---|---|
| Jessie Aguilar | EPCC | Mathematics | Feb. 2000 – Oct. 2000 |
| Liza Aguirre | CLISD | Horizon High School | Oct. 2004 - present |
| Nancy Arroyo | YISD | Riverside High School | Oct. 2003 - present |
| Alicia Beltran | SISD | Sanchez Middle School | Oct. 2001 – present |
| Patricia Benitez | EPISD | Magoffin Middle School | Oct. 2004 - present |
| Naomi Berglund | EPISD | Mesita Elementary School | Feb. 2000 - April. 2001 |
| Vicky Brown | SISD | Helen Ball Elementary School | Feb. 8 - July 2000 |
| Lupe Bujanda | EPISD | Bowie HS; MSP Staff Dev | Feb. 2000 - present |
| Lien Diaz | EPISD, ESC R19 | Mentor MS, MSP Staff Dev | Oct. 2001 - present |
| Art Duval, Ph.D. | UTEP | Mathematics | Feb. 2000 - present |
| Pat Estrada | YISD | Mentor – MS | Feb. 2000 – June 2004 |
| Maritza Fernandez | YISD | Hacienda Heights Elem. School | Oct. 2002 – June 2004 |
| Carol Gardner | EPISD | USP Mentor - Elementary School | Feb. 2000 – June 2003 |
| Ann Gates, Ph.D. | UTEP | Engr. - Computer Science | Feb. 2000 – June 2002 |
| Sandra K. Garza | SISD | Mentor ES, SISD Elem Math | Feb. 2000 – present |
| Joanne Gillis | EPISD | Franklin High School | Feb. 2000 – June 2003 |
| Terrie Giron | YISD | Mentor HS, MSP Staff Dev | Feb. 2000 – present |
| Martha Gonzalez | EPISD | Math Leader,Vilas E S | Oct. 2004 - present |
| Margie Gutierrez | SISD | Mentor MS | Feb. 2000 – June 2003 |
| Carol Hardee | SISD | Mentor MS | Oct. 2002 - present |
| Greg Hatch | SISD | MSP Staff Developer | Oct. 2003 - present |
| Veronica Hernandez | EPISD | Mentor HS, MSP Staff Dev | Feb. 2000 – June 2002 |
| Helmut Knaust, Ph.D. | UTEP | Mathematics | Oct. 2004 - present |
| Martha Kaudaissy | SISD | Campestre Elementary | Oct. 2001 – present |
| Blanca Lopez-Martinez | YISD | Mentor ES | Feb. 2000 – June 2003 |
| Tony Murillo | SISD | Socorro Middle School | Oct. 2002 - present |
| Becky Ontiveros | EPISD | Mentor MS | Feb. 2000 - Aug. 2001 |
| Jaime Ortiz | YISD | Parkland High School | Feb. 2000 - June 2000 |
| Debra Paulson | EPISD | Hornedo MS, Math Program Asst. | Feb. 2000 – present |
| Joanne Peeples, Ph.D. | EPCC | Mathematics | Oct. 2002 – present |
| Estella Quinones, Ph.D. | UTEP | Metallurgical & Materials | Oct. 2002 - present |
| Martin Rede | SISD | Mentor HS, MSP Staff Dev | Feb. 2000 – present |
| Diane Reed | YISD | J. M. Hanks High School | Feb. 2000 - present |
| Ullrich Reichenbach | SISD | Montwood High School | Feb. 2001 – Dec. 2002 |
| Fred Rojas | SISD | Americas High School | Oct. 2002 – June 2004 |
| Edna Salas | SISD | Hilley Elementary School | Oct. 2002 - present |
| James Salazar | YISD | Bel Air HS, MSP Staff Dev | June 2001 – present |
| Gabriela Schwab | EPCC | Mathematics | Oct. 2002 - present |
| Marsha Self | EPCC | Mathematics | Feb. 2000 – June 2003 |
| Gus Serrano | YISD | Ranchland Hills Middle School | Feb. 2000 – April 2003 |
| Diane Seufert | EPISD | Carlos Rivera Elementary | April 2001 – present |
| Mariano Silva | EPISD | Mentor MS | June 2003 |
| Sue Spotts | EPISD | Wiggs Middle School | Oct. 2000 - present |
| Mourat Tchoshanov, Ph.D., | UTEP | Mathematics | Feb. 2000 - present |
| Rita Tellez | EPISD | Bowie HS; HS Math Facilitator | Oct. 2002 – Oct. 2004 |
| Tom Ukestad | SISD | Socorro High School | Feb. 2000 - present |
| Jaime Vasquez | SISD | Hueco Elementary School | Feb. 2000 - May 2000 |
| Donnett Vollmer | EPISD | Magoffin Middle School | Feb. 2000 - May 2000 |
| Xiaomin Wang, Ph.D. | EPCC | Mathematics | Oct. 2004 – April 2005 |
| Matthew Winsor, Ph.D. | UTEP | Mathematics | Oct. 2004 – present |
| Stella Woo | EPISD | Silva Magnet High School | Oct. 2004 – present |
| Lucy Hernandez. Michal | Director, K-16 Mathematics Alignment Initiative<br>MSP Director of Mathematics and Science | | Jan. 2000 - present |

# PRECALCULUS COURSE OUTLINE

## I. COURSE DESCRIPTION

Precalculus provides students with a strong foundation and understanding of both algebraic and transcendental functions. Students systematically work with functions and their multiple representations, strengthen their algebraic knowledge base of real numbers, and use both functions and real numbers with fluency to solve problems. Precalculus extends how students view, interpret, and apply functions, and bridges concepts of algebra and geometry with calculus. Utilizing real-life applications, the course prepares students for higher level mathematics and other courses in physics, engineering and chemistry, and will prepare students for fields of study including astrophysics, engineering, medical fields, research science, statistics, and teaching.

## II. PREREQUISITE KNOWLEDGE

Students entering Precalculus should have successfully completed Algebra II and should know and be able to use:

A. Foundations and basic knowledge of functions
B. Dependent and independent variables
C. Domain and range of functions
D. Function notation
E. Rectangular coordinate system
F. Relationship and connection between and multiple representations of functions (table, graph, models, algebraic)
G. Communicate using functions
H. Manipulate functions and equations
I. Right triangle properties
J. Similar triangles
K. Pythagorean Theorem
L. Geometry
M. Solve equations and simple inequalities
N. Model problems using functions
O. Multiply and factor algebraic expressions
P. Understand rational, real, and imaginary numbers
Q. Roots and exponents
R. Parameter changes and their effect on functions
S. Underlying processes – tools, investigation, connections, communication

## III. CONTENT

In Precalculus the focus will be on learning and being able to use:

A. Foundations of algebraic, transcendental, and trigonometric functions
B. Continuity of functions
C. Trigonometric properties and identities
D. Radian and degree measure and notation
E. Conic sections
F. Using functions to solve, analyze, and make predictions
G. Synthetic division
H. Understand, visualize, and predict patterns of algebraic and geometric sequences

I. Limits of sequences, series
J. Asymptotes, end behavior, and critical points to sketch graphs of functions
K. Analysis and description of functions as increasing, decreasing, symmetric, containing roots, etc.
L. Relationship of the natural logarithm with the irrational number e
M. Model and solve physical situations using vectors
N. Piecewise functions
O. Solve problems involving real life situations using functions
P. Given two or more functions, obtain a third function by adding, subtracting, multiplying, and dividing them

## IV. ASSESSMENT

A. It is suggested that a variety of methods be used to assess student learning. This includes assessments that show student work as well as student explanations of their work. These assessments might include both traditional and alternative methods such as:
   1. Performance based tasks
   2. Open book (including homework)
   3. Technology-based presentations
   4. Interviews
   5. Observations
   6. Portfolios
   7. Projects with rubrics (individual and group)
   8. Warm-up quizzes
   9. Multiple choice
   10. Open response
   11. Comprehensive, multi-step problems
   12. Final Exam – The final exam should be a comprehensive exam standardized by campus with future plans to standardize by district, city, and/or state. Having all students taking a final exam will prepare students for college final exams. The final exam should count approximately 25% of the grade.

B. Recommended Course Grade – Each district has guidelines for courses grades and, whenever possible, it is suggested that final course grades for students be guided by the following:
   1. Formative assessments (daily tools such as warm-ups, quizzes, teacher observations and interviews, group work) ------ 25%
   2. Closed book assessments (Open response, multiple choice, quantitative comparisons, SAT problems, multi-step problems) ------ 25%
   3. Open book assessments (homework, projects, presentations, portfolios) ----- 25%
   4. Final Comprehensive Exam over the entire course content ----- 25%

## V. TIMELINE

A brief overview of algebra may be given during the first week of the semester. It is recommended that the rest of the time should be allotted as follows and that any further review given be embedded in the following units as needed.

A. Functions (to include modeling, composition, operations). 70%
   Polynomial, Rational, Radical, Piecewise (20%)

Exponential, Logarithmic (15%); Trigonometric (35%)

B. Sequences and Limits, 10%

C. Conic Sections, 10%

D. Vectors, 10%

## VI. INFORMATION/RESOURCES

A. FOR STUDENTS

1. Course description
2. Teacher information (conference period, office hours)
3. Work, projects, homework, exams, etc., to be produced by the students including grading policy for each
4. Rubrics for projects/presentations/portfolios
5. Resources – tutoring, lab, Internet web sites specific to the course, computer programs, teacher conference period, other outside support available
6. Weekly calendar
7. Materials: It is recommended that a textbook and graphing calculator be issued to each student.

B. FOR TEACHERS

1. Labs: math and computer
2. Materials: textbooks, graphing calculators with view screens, charts, transparencies, etc.
3. Computer: hardware, software, and multi-media resources
4. Professional Networks: provisions for teacher teaming during conference time, professional development/credits or endorsements to increase salaries, peer coaching
5. References: instructor manuals, journals, Educational Resource Information Clearinghouse, Internet websites
6. CBL- Computer Based Lab and CBR – Computer Based Range
7. Vertical alignment information on K-16 alignment initiatives
8. Suggested course calendar

## VII. MATRIX MAPPING TOPICS TO COGNITIVE DEMANDS

A. Attached is a matrix that matches cognitive demands to knowledge and skills in Precalculus. Using cognitive demands has been guided by the work of Andrew Porter, Norman Webb, and John Smithson. The cognitive demands identified by Porter, Webb, and Smithson were used as models and modified by the working group to fit the work in Precalculus. These identify thinking levels that incorporate five (5) levels of cognitive demands. They are listed on the matrix from higher order to lower order as you read from left to right. Frameworks also map the course with state and national mathematics standards and, for some courses, frameworks also map textbook and materials used in major independent school districts.

B. Cognitive Demands for Mathematics

Cognitive demands assist teachers in distinguishing what a student is expected to know and be able to do with mathematics content and what level of thinking students must be engaged in while learning the content. This mapping of topics to cognitive demands describes content knowledge that will not merely be stored, but also understood, represented, organized, connected, and structured in ways that facilitate retrieval and application of knowledge. With knowledge and skills mapped to cognitive demands, teachers know how to engage students to use, represent and connect pieces of content knowledge in coherent ways that will determine whether students understand knowledge deeply and can use it to solve new problems. The cognitive demands are not linear nor are they sequential. In many instances they overlap and are not clearly separated. They are to:

1. **Generalize –** make and prove conjectures, prove statements generate questions
2. **Make Connections –** transfer knowledge, connect two or more concepts to solve non-routine problems
3. **Understand Concepts –** communicate "big ideas", justify and explain solutions to problems, use multiple representations to model mathematical ideas, select the most appropriate representation for given situations
4. **Perform Procedures –** do computations, make observations, measure and compare, solve routine problems
5. **Memorize –** facts, definitions, formulas, properties, rules

C. Format and Further Information on Matrix Structure
1. All TEKS are included in the frameworks.
2. Items in the matrix appearing in regular fonts are TEKS and are placed within the appropriate cognitive demand.
3. Italicized items are used:
   a. to support the teaching and learning of a topic; these do not reference a TEKS;
   b. to paraphrase a TEKS to address the different levels of cognitive demands; these will have a referenced TEKS and are placed under multiple cognitive demands (e.g. p. 12, TEK cP.2A)
4. Strands/topics in matrices overlap and may be integrated
5. Cognitive demands overlap and are not linear
6. The framework is not intended to be sequential.
7. Other topics supporting the study of precalculus may be included in the matrix such as: co-terminal, reference, and initial angles; initial and terminal sides; radian measure; arc-length, area of a circular segment; and circular chord.z

Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| **Knowledge and Skills** | **Cognitive Demands** | | | | |
|---|---|---|---|---|---|
| | **Generalize** | **Make Connections** | **Understand Concepts** | **Perform Procedures** | **Memorize** |
| cP.1 The student defines functions, describes characteristics of functions, translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. | cP.1E Investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically | cP.1D Recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc) points on the graph of a function, and the symbolic representation of a function<br><br>*cP.1A Model real world situations by the appropriate function* | cP.1A Describe parent functions symbolically, graphically, *[and verbally]* including:<br>$f(x) = x^n$ $f(x) = \ln x$<br>$f(x) = e^x$ $f(x) = \log_a x$<br>$f(x) = 1/x$ $f(x) = \lvert x \rvert$<br>$f(x) = a^x$ $f(x) = \sin x$<br>$f(x) = \arcsin x$, etc.<br><br>*cP.1B Describe what situations cause restrictions of domain and range within certain parent functions*<br><br>cP.1C Describe symmetry of graphs of even and odd functions<br><br>*cP.1D Recognize significant features of a function, the points on the graph of a function, and the symbolic representation of a function*<br><br>cP.1E *Describe the conditions that result in the existence of asymptotes* | cP.1B Determine the domain and range of functions using graphs, tables and symbols<br><br>cP.1D *Find zeros, maxima, and minima of functions*<br><br>*cP.1E Find the equations of asymptotes* | *Vocabulary:*<br>*Asymptotes*<br>*Points of Discontinuity*<br>*Sine, Cosine, Tangent, Cotangent*<br>*Secant, Cosecant,*<br>*Arcsine, Arccosine*<br>*Arctangent, Arccotangent*<br>*Arcsecant, Arccosecant* |

| Timeline | Textbook and Materials | NCTM Standards |
|---|---|---|
| | | Model problem situations with objects and use representations such as graphs, tables and equations to draw conclusions<br>Investigate how a change in one variable relates to a change in a second variable |

# Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| **Knowledge and Skills** | **Cognitive Demands** | | | | |
|---|---|---|---|---|---|
| | **Generalize** | **Make Connections** | **Understand Concepts** | **Perform Procedures** | **Memorize** |
| cP.2 The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. | cP.2C Investigate identities graphically and verify them symbolically, including logarithmic and exponential properties and trigonometric identities | *cP.2A* Model a given algebraic representation<br><br>cP.2A *Use transformations of functions to model a given situation* | *cP.2A Explain and justify how transformations and combinations of transformations affect the graph, table, formula, domain, range and intercepts of the parent function, including a f(x), f(x) + d, f(x-c), f(bx), \|f(x)\|, f(\|x\|)*<br><br>*cP.2A Given a verbal or graphical description of a transformed function, determine the algebraic representation of the function*<br><br>*cP.2B Given the results of the composition of two or more basic functions, find the basic functions that were composed*<br><br>*cP.2B Explain the composition of functions, finding inverses, verbally, numerically, symbolically, and graphically* | cP.2A Apply basic transformations including:<br>a f(x)<br>f(x) + d<br>f(x-c)<br>f(bx)<br>and compositions with absolute value functions, including \|f(x)\|, and f(\|x\|) to the parent function<br><br>*cP.2B Perform operations including composition on functions and find inverses of functions* | *Vocabulary:*<br>*Composition of functions*<br><br>*Inverse functions*<br><br>*Transformations*<br><br>*Parent functions*<br>*Polynomial*<br>*Trigonometric*<br>*Exponential*<br>*Trigonometric*<br>*Absolute value*<br><br>*Vertical Expansion*<br>*Horizontal Expansion*<br>*Vertical Contraction*<br>*Horizontal Contraction*<br>*Vertical Dilation*<br>*Horizontal Dilation* |

| | Textbook and Materials | NCTM Standards |
|---|---|---|
| | | Understand and perform transformations such as arithmetically combining composing and inverting commonly used functions<br>Use technology to perform such operations on more complicated symbolic expressions<br>Understand patterns, relations and functions<br>Represent and analyze mathematical situations and structures about algebraic symbols |

Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| **Knowledge and Skills** | **Cognitive Demands** | | | | |
|---|---|---|---|---|---|
| | **Generalize** | **Make Connections** | **Understand Concepts** | **Perform Procedures** | **Memorize** |
| cP.3 The student uses functions and their properties, tools and technology, to model and solve meaningful problems.<br>. | cP.3A Investigate properties of trigonometric and polynomial functions<br><br>cP.3D Use properties of functions to analyze and solve problems and make predictions | cP.3C Use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient)<br><br>*cP.3E Solve real-life problems that describe physical phenomena using trigonometric functions* | cP.3B Use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data<br><br>*cP.3D Use properties of functions to solve problems*<br><br>*Determine which parent function models a given situation* | cP.3E Solve problems from physical situations using trigonometry, including the Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed | *Graph of parent functions*<br>*Properties of parent functions*<br><br>*Vocabulary:*<br>*Trigonometric ratios*<br>*Reference angles*<br>*Radian representation of reference angles*<br>*Law of Sines*<br>*Law of Cosines*<br>*Area formulas*<br>*Co-terminal*<br><br>*Know multiple notations, i.e arcsine (x) = $sin^{-1}(x)$* |

| Timeline | Textbook and Materials | NCTM Standards |
|---|---|---|
| | | Understand and compare properties of classes of functions including exponential, polynomial, logarithmic and periodic functions<br>Use mathematical models to represent and understand quantitative relationships<br>Analyze change in various contexts<br>Connections p. 354, Reasoning and proof p. 342, Algebra p. 296, Geometry p. 308 |

Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| Knowledge and Skills | Cognitive Demands | | | | |
|---|---|---|---|---|---|
| | **Generalize** | **Make Connections** | **Understand Concepts** | **Perform Procedures** | **Memorize** |
| cP.4 The student uses sequences and series, as well as tools and technology, to represent, analyze, and solve real-life problems. | *Derive a general formula for a sum of finite powers of integers e. g.*<br>$1 + 2 + 3 + \ldots + n =$<br>$1^2 + 2^2 + 3^2 + \ldots + n^2 =$<br>$1^3 + 2^3 + 3^3 + \ldots + n^3 =$<br>… | cP.4B Use arithmetic, geometric, and other sequences and series to solve real-life problems<br><br>cP.4D Apply sequences and series to solve problems including sums and binomial expansion<br><br>Connect decimal notation and geometric series | cP.4C Describe limits of sequences and apply their properties to investigate convergent and divergent series<br><br>*Use multiple representations to illustrate a limit of a sequence/series*<br><br>cP.4C *Apply properties to investigate convergent and divergent series* | cP.4A Represent patterns using arithmetic and geometric sequences and series<br><br>cP.4D *Expand binomials raised to the nth power i.e.* $(a + b)^n$<br><br>*Find the general term for an arithmetic or geometric sequence given at least 5 terms*<br><br>*Find terms of a sequence the* $n^{th}$ *term*<br><br>*Find* $n^{th}$ *term of a sequence given information about the sequence*<br><br>*Find the sum of finite and infinite geometric series* | *Vocabulary:*<br>*Sequence*<br>*Arithmetic sequence/series*<br>*Geometric sequence/series*<br>*Summation notation*<br>*Finite sequence/series*<br>*Infinite sequence/series*<br>*Recursive sequence*<br>*Arithmetic sequence (AS)*<br>*Geometric sequence (GS)*<br>*Fibonacci sequence*<br>*Harmonic sequence*<br>*Limit*<br>*Convergent sequences/series*<br>*Divergent sequences/series*<br>*Sum formula for convergent geometric series* |

| Timeline | Textbook and Materials | NCTM Standards |
|---|---|---|
| 10% | | Generalize patterns using recursively defined functions<br>Use a variety of symbolic representations including recursive equations for functions and relations<br>Use symbolic relations to represent relationships arising from various functions |

 

Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| Knowledge and Skills | Cognitive Demands | | | | |
|---|---|---|---|---|---|
| | **Generalize** | **Make Connections** | **Understand Concepts** | **Perform Procedures** | **Memorize** |
| cP.5 The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations. | *Make a conjecture about what happens to the graph of the conic section as you change the constants in* $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ | cP.5A Use conic sections to model motion such as the graph of velocity vs. position of a pendulum and motions of planets<br><br>cP.5D Use parametric functions to simulate problems involving motion.<br><br>*Connect the equations of conic sections to the intersection of a plane with a cone* | cP.5B Use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound<br><br>cP.5C Convert between parametric and rectangular forms of functions and equations to graph them<br><br>*Given the graphic representation of a conic, write the equation and describe the features of the conic section.* | *Use completing the square to find vertices, foci, and other defining features of a conic section*<br><br>*Recognize the type of conic section (ellipse, hyperbola, parabola, circle , etc.) from the general conic equation or the standard equation* | *Vocabulary:*<br>*Ellipse*<br>*Hyperbola*<br>*Parabola*<br>*Circle*<br>*Parametric equation*<br>*Locus of points*<br>*Eccentricity*<br>*Major/minor axis*<br>*Focus/foci*<br>*Directrix*<br>*Latus Rectum*<br>*Properties of each conic section* |

| Timeline | Textbook and Materials | NCTM Standards |
|---|---|---|
| 10% | | A variety of symbolic representations including parametric equations for functions and relations |

Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| **Knowledge and Skills** | **Cognitive Demands** | | | | |
|---|---|---|---|---|---|
| | **Generalize** | **Make Connections** | **Understand Concepts** | **Perform Procedures** | **Memorize** |
| cP.6 The student uses vectors to model physical situations. | *Make a conjecture about the addition of vectors using magnitude and direction from the initial point of the first vector to the terminal point of the last vector for any dimension n* | cP.6B Analyze and solve *non-routine* vector problems generated by real-life situations vector problems | cP.6A Use the concept of vectors to model situations defined by magnitude and direction<br><br>*cP.6A Represent physical situations using vectors*<br><br>*cP.6A Given a vector diagram describe the situation it represents*<br><br>*Explain relationship between components of a vector and its graphical representations*<br><br>*Explain addition and subtraction of two vectors using triangle or parallelogram laws and component form*<br><br>*Explain the relationship between dot products and perpendicular vectors* | *Add and subtract two vectors*<br><br>*Construct a vector given magnitude and direction and/or initial and terminal points*<br><br>*Decompose a vector into its components*<br><br>*Multiply a vector by a scalar using graphs and component form*<br><br>*Compute dot products*<br><br>*Optional: Compute cross and triple scalar product* | *Vocabulary:*<br>*Vector*<br>*Initial point*<br>*Terminal point*<br>*Magnitude of vector*<br>*Direction of vector*<br>*Components of a vector*<br>*Resultant vector*<br>*Unit vector*<br>*Zero vector*<br>*Properties of vectors*<br>*Analytic form of vectors*<br>*Dot product*<br>*Cross product*<br>*Initial point*<br>*Terminal point*<br>*Parallelogram law* |

| Timeline | Textbook and Materials | NCTM Standards |
|---|---|---|
| 10% | | Apply transformations and use symmetry to analyze mathematical situations<br>Understand vectors and matrices as systems that have some of the properties of the real number system<br>Develop understanding of properties of and representations for the addition and multiplication of vectors and matrices<br>Develop fluency in operations with vectors |

Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

| Type of Assessment | Purpose of Assessment | How often? | Materials Needed | Descriptors for Acceptable Level of Performance |
|---|---|---|---|---|
| **FORMATIVE ASSESSMENTS 25%** | | | | |
| **Student/Teacher Interviews** | **To examine the thinking process of students** | **Weekly** | **Paper/recorder** | **Vocabulary, participation** |
| **Observations Discussions** | **To measure if a student is able to communicate understanding of a concept** | **As often as possible** | **Checklist** | **Participation** |
| **Warm-up** | **Daily review reinforcement** | **Daily** | **Overhead paper/pencil calculators** | **Working problem 70% or better** |
| **CLOSED BOOK ASSESSMENTS 25%** | | | | |
| **Multiple Choice Exams** | **Evaluate skills**<br>Preparation for mandated tests | **Twice a week** | **Scanners**<br>Multiple tests | **80% correct** |
| **Open Response Exams** | **Test individual student understanding** | **3 - 4 per grading period** | **Rubrics**<br>Test | **80% correct** |
| **OPEN BOOK ASSESSMENTS 25%** | | | | |
| **Performance Based Task** | **To measure how close student are in mastering TEKS standards**<br>To measure how well the student transfers and integrates knowledge<br>Measure understanding of concepts | **Every 6 – 12 weeks** | **Tools on a student generated list**<br><br>Calculator, Computer, chart paper, poster board, transparencies, presentation tools, manipulatives, video equipment, multi-media | **Rubric (descriptor) based on TEKS or standard**<br><br>Include:<br>Content criteria,<br>Process criteria<br>Presentation criteria |
| **Presentation in groups of two, three, or four** | **Summative** | **1 per grading period** | **Research material**<br>Access to media center<br>Consumable material | **Knowledge of content**<br>Vocabulary<br>Oral communication |
| **Homework and "Open book" exams** | **Student finds and uses information in resources to:**<br>▪ Solve problems and explain solutions<br>▪ Explain mathematic concepts<br>▪ Prepare for "closed book" exam | **Midway thru and at the end of a "Big Idea"** | **Textbooks, notes, library, computer resources, calculator, manipulatives** | **Demonstrate knowledge and understanding of the big idea at the "Mastery Level"** |
| **Technology-based presentation** | **Extend understanding of concepts** | **2 per semester** | **Computer software, calculators** | **Student/teacher created rubric** |
| **Journaling** | **Thinking process, communication** | **Weekly** | **Notebook paper, index cards 5x7** | **Clear writing about topic, turning it in** |
| **Projects** | **Extension of concepts**<br>Tests different styles of understanding | **Every 6 weeks** | **Varies with written rubric describing project** | **Rubric Requirements** |
| **FINAL COMPREHENSIVE EXAM 25%** | | | | |
| **Comprehensive** | **To measure what student knows and is able to do with the knowledge acquired from the entire course** | **1 at the end of the entire course** | **Test** | **80% correct** |